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Elliott Lieb and the Art of Mathematical Physics

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For many distinguished scientists the seventieth birthday is a welcome opportunity to enjoy the fruits of past achievements and rest on laurels. Not so for Elliott Lieb who continues to work at an undiminished pace (the famous 12 hour day, 7 days a week), writing one paper after another full of deep insights and exciting new results. His output is impressive just by its sheer quantity which is rare in mathematical physics: His publication list has close to 300 items to date, including well over 200 original research papers, many review articles and two books. His Selecta of papers on Stability of Matter, now in the third edition, has over 800 pages, another volume on Inequalities more than 700, and two more volumes are in preparation. The number of his collaborators to date is over 80.

But even more important than such quantitative measures is the quality of his work in the many disciplines where he has made lasting contributions. A characteristic for his working style is a combination of formidable mathematical skills with good taste. He always treats important and difficult problems, more often than not so extensively that it can take a long time until an improvements is found, and then often by himself. But one of the many things his younger collaborators learn from him is that not every difficult problem is worth working on; the selection of problems is an integral part of the art of mathematical physics. His papers also distinguish themselves by the thoroughness of the arguments in accord with another rule he imprints on his collaborators: Write everything in such a way that you can explain it to your grandchildren in due time.

When Elliott wrote his first research papers in the mid 50's, modern mathematical physics in the sense the term is used now hardly existed. Classical mathematical physics, including mechanics and dynamics of continuous media, was, of course, a venerable discipline (as it still is today) but

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a frontal attack on problems in quantum theory and statistical physics with the tools of hard mathematical analysis was by no means an obvious thing to do. Elliott, like most other pioneers of modern mathematical physics of these days, did not come to the subject with the background of a mathematician but that of a theoretical physicists. These pioneers, to which I also count two other great mathematical physicists I have had the good fortune to know well, Hans Jürgen Borchers and Walter Thirring, came to their sophisticated methods by realizing that rigorous mathematics was needed to obtain unambiguous answers to the physical questions they were asking.

Here is a brief list of some of Elliott Lieb's major fields of research: Models of statistical mechanics, especially exactly soluble models, many-body quantum physics, including the Bose gas and Bose–Einstein condensation, entropy inequalities, the quantum theory of Coulomb systems and exact results on atoms and molecules, stability of matter, matter in strong magnetic fields, harmonic maps and liquid crystals, quantum electrodynamics, the second and the third law of thermodynamics. It is not possible here to do proper justice to the wealth of results Elliott has obtained in these fields during his long career. For this I refer instead to the Selecta volumes already mentioned and their introductions. But there is one topic that I would like to say a little more about from my personal experience, our joint work on the second law of thermodynamics. I know that this is also a subject that is dear to Elliott's heart.

The physics curricula at most universities include a joint course on thermodynamics and statistical mechanics. There are, however, basically two different ways the subject is presented to students. The first is to start with thermodynamics, usually stating its Laws in the traditional verbal terms and then introducing entropy and absolute temperature using concepts like heat, empirical temperature scales and Carnot cycles. In this approach statistical mechanics enters at a later stage. The second approach is to start with Boltzmann's famous formula for entropy, making its basic properties plausible by computing it for a few idealized examples and from then on mixing thermodynamical and statistical mechanical arguments. Judging from textbooks on the subject the second approach enjoys much popularity nowadays and many students are likely to regard thermodynamics as a mere corollary to statistical mechanics.

When Elliott and I started our discussion about thermodynamics in the early 90's it soon became clear that we both shared the admiration for the Second Law and had the firm conviction that such a perfect and unbreakable principle of nature (within its range of applicability) deserves a solid and unambiguous foundation, independent of hard to define concepts like heat, empirical temperatures and idealized Carnot cycles, but also independent of statistical mechanics. Since Elliott is a major player in modern statistical mechanics and bearer of the Boltzmann medal, I must admit that the last statement from Elliott surprised me at first. But he soon convinced me that although he values statistical mechanics as much as any physicist, it is simply a fact that a derivation of the Second Law from statistical mechanical principles alone is still beyond anyone's computational ability (except in some idealized situations). On the other hand we were both not satisfied with the traditional approach to the law that involve a myriad of idealizations and unstated assumptions.

We came back to our discussion on thermodynamics several times during the following years but rather as a pastime occupation during breaks from other work. Things started to become serious when we discovered that it is possible to characterize entropy completely in terms of the relation of adiabatic accessibility between equilibrium states and that there is a simple formula for it that does not involve Carnot cycles nor any other of the traditional concepts. This work culminated in a long paper that appeared in Physics Reports in 1999. A shorter summary had already appeared in the Notices of the American Mathematical Society in 1998 and in 2000 a summary for physicists appeared in Physics today.

It was only after our work was completed that we became aware of Robin Giles' great book from 1964 on the Mathematical Foundations of Thermodynamics that approaches the subject in a spirit similar to ours. The fact that we did our work unaware of Giles can be regarded as a testimony for the naturalness of this approach. But there are also marked differences. Most importantly, a basic axiom of Giles about the comparability of states with regard to the relation of adiabatic accessibility is a theorem in our work. Its proof is based on an interplay of convexity and analytical arguments, both topics where Elliott's great experience and skills were invaluable. The final chapter of our thermodynamical work concerns mixing and chemical reactions. Here the ambition was to do entirely without gedankenexperiments involving nonexistent semipermeable membranes that are essential in the traditional approaches. We succeeded but it took a whole year. I found Elliott's persistence most admirable and without it this part would never have been completed. Altogether this thermodynamical project was one of the most enjoyable episodes of my scientific life and I had the definite impression that Elliott enjoyed it just as much.

Elliott's scientific achievements have been acknowledged by many honors, including the Dannie Heineman Prize of the American Physical Society, the Boltzmann medal of the International Association of Pure and Applied Physics, the Max Planck Medal of the German Physical Society, the Schock Prize of the Royal Swedish Academy, the Levi L. Conant Prize of the American Mathematical Society, the Henri Poincaré Prize of the International Association of Mathematical Physics and honorary doctorates from Lausanne, Copenhagen and Munich. During the symposium that was held in Vienna around his birthday on July 28 2002 and attended by well over 100 friends and colleagues from all over the world, the Austrian Minister of Education, Science, and Culture awarded him the highest Austrian medal for Science, the *goldenes Ehrenzeichen für Wissenschaft und Kunst*. Another conference in his honor was held at Rutgers University in December 2002.

In addition to his scientific work, Elliott has been of invaluable service to the community of mathematical physicists in other ways. He has twice been president of the International Association of Mathematical Physics and and has served on various committees of the American Mathematical Society. His engagement was of great importance for the foundation of the Erwin Schrödinger Institute for Mathematical Physics in Vienna eleven years ago and he is a member of its International Advisory Board. But his most important service, perhaps, is to set, by his work, the gold standard by which future work in mathematical physics will be measured.